Prerequisite Skills
and
Mathematics Learning

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Prerequisite Skills and Mathematics Learning[1]

Toys and games are synonymous with play, pleasure, and relaxation. Almost everyone likes to play and, in one form or other, this continues throughout one's life. But play is not just filling an empty period or a leisure activity. It is also an important learning experience—essential for growth and development. For example, babies play with their fingers and toes and in so doing bring about a social interaction with adults who join in their game. As the baby explores this form of play, the child acquires the prerequisite skills needed to learn numeracy.

In addition to being a means of relaxation, play is seen as a means to work off aggression, to learn basic survival skills (as is also observable in the animal kingdom), and to learn social behavior (competitive and cooperative). But more importantly, the role of play is to engage in learning and to gain interest in learning, especially as children enter school. During Pre-Kindergarten and Kindergarten, the purpose of early childhood experiences like games is to develop:

(a) Neuro-psycho-physiological maturation,
(b) Socio-linguistic maturation,
(c) Quantitative reasoning, and
(d) Spatial orientation and space organization.

Development in all of these areas is facilitated by formal and organized learning experiences, but formal and informal play, toys and games also have an important role.

Fun is a great motivator for learning any subject, even mathematics. Unfortunately, many in our society feel inadequate even when it comes to everyday calculations, and we view mathematics as an anxiety-provoking task. That makes it difficult for teachers to teach mathematics and for students to invest the interest, time, and energy it takes to learn mathematics. Fortunately, though, games and toys offer motivation and fun to foster learning.

Most children come to Kindergarten able to recite the alphabet fluently, in the case of English language, the twenty-six letters. In many countries that number is much larger. Although this is just a rhyme for children at this stage, it is the beginning of the development of language containers for sounds and reading. However, few children enter able to

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[1] Excerpted from
Games and Their Uses in Mathematics Learning (Sharma, 2008/2012)

recite the numbers up to thirty in sequence. To prepare children for school, parents need to pay the same kind of attention to numbers as they do to letters. In many countries, parents do often chant a rhyme such as the one below while touching each of the child's toes:

one, two, three, four, five,
once I caught a fish alive:
six, seven, eight, nine, ten,
then I let him go again.

This is the beginning of parents' efforts to assist the child to learn to count. But this effort needs to go beyond just the first ten numbers. And in the information and Internet age, smart-phones, hand-held devices of various kinds, computers and web-based games, already a source of play among children, can be used to teach numbers higher than ten.

In addition to teaching numbers, these devices and games on them, provide opportunities to develop many learning skills (visuo-spatial, sequencing, and spatial sense) as well as sociolinguistic skills. In addition, multi-sensory games develop and integrate tactile/kinesthetic, visual, auditory, socio-linguistic, and executive function skills.

These days, board games are not limited to play dates and family game nights. Classic games like Scrabble, Candy Land, Go Fish, and Sorry are finding their way into classrooms as educators use many of these popular games to reinforce mathematics, language and critical thinking skills. Numerous research studies support the assertion that playing board games helps students improve mathematics and thinking skills. For example, in one study, disadvantaged preschool students played a simple numeric board game four times for 15-20 minutes at a time over a two-week period. At the end of the two weeks, researchers found students' knowledge of math greatly increased in several different areas related to quantitative thinking and numbersense. Numbersense is a form of quantitative thinking—for example, knowing what a collection representing the number 5 looks like and knowing that 5 is less than 8 and that 8 is made up of 5 and 3. That knowledge does not come just through counting.

Number concept and numbersense depend on counting, learning quantitative language, and understanding the spatial representation and distribution of objects in the representation of a number. While counting is the beginning of the number conceptualization process, it takes several developmental steps to attach a number value to collections and amounts, give meaning to numbers and reach an understanding of the interrelationships of numbers.

Number concept is the integration of several prerequisite skills: acquiring number names, sequencing, one-to-one correspondence, visual clusters—arrangement of objects and decomposition/re-composition of visual clusters (numbers)—can the child break the number (from 2 to 10) into its component sub-clusters (smaller numbers) (e.g., 10 is made up of 1 and 9; 2 and 8; 3 and 7; 4 and 6; and 5 and 5).

Number conceptualization is easily achieved by games and toys. For example, games involving playing cards, dominoes, or dice bring together the essential skills—sequencing, one-to-one correspondence, visual clustering, and decomposition/recomposition. Many card and board games reinforce number concept and numbersense and help children learn these pre-requisite skills, but most importantly they develop logical reasoning and the communication of ideas.

The benefits of board games are not limited to mathematics. They can build vocabulary, spelling, and logical reasoning skills. Basically any game can be adapted to help learn. For example, the card game ***Go Fish*** can be modified into a game ***Go Make Ten, Go Make Eleven,*** and ***Go Make ____***, etc. This is a series of games to develop the 45 ***sight facts*** (the sums of two numbers up to ten). It is played in the same way as **Go Fish**, but here children make as many pairs of two cards that add to ten (or any other number) to practice the sight facts. In a short time, children learn the most important arithmetic fact clusters: what two numbers make ten? Children who have not mastered this family of addition facts up to 10 have difficulty mastering other addition facts (a non-negotiable skill at first grade). The game can further be modified to other number facts: ***Go Make Nine***, ***Eight***, etc. Because of their intrinsic entertainment value, board games provide educators with an effective tool for engaging students. Games facilitate a welcoming learning atmosphere because students think they're just having fun.

Many parents want to work with their children to help them in mathematics, but they may feel that they have limited mathematics training and understanding of mathematical concepts, particularly "modern math" or "new ways" of teaching mathematics. Other parents may be impatient with children having problems because they have high expectations of their children. While working with their children, they may also be mixing the roles of parenting and teaching, which can cause difficulties both in learning and personal relationships. To avoid transferring their own anxieties or setting unrealistic expectations, parents should work on mathematics only if they feel comfortable with mathematics and have realistic goals for their children.

Despite these possible limitations, there is a great deal parents can do to help children in learning mathematics. For example, they can help children acquire prerequisite, non-mathematical, skills for mathematics learning: 1) Memory: to learn basic terminology and hold information in their mind's eye (working memory, visualization), 2) Inductive thinking: to see patterns, i.e., going from specific examples to generic rules, 3) Deductive thinking: to apply general rules to specific problems and situations, 4) Spatial orientation/space organization (relational words, such as: close to me, to my left, above me, below the table, under the plate, etc.), and 5) Task Analysis: to break down a given/larger problem into smaller, manageable, solvable problems. Other prerequisite skills help children learn, retain, and master formal concepts, skills, and procedures in mathematics:

- ***Matching/one-to-one correspondence with sequence***
- ***Classification/class inclusion***
- ***Visual clustering***
- ***Ordering and sequencing***
- ***Visualization***
- ***Ability to follow sequential directions***
- ***Spatial orientation and space organization***
- ***Estimation***
- ***Pattern recognition, extension, and its application***
- ***Deductive and inductive thinking***

Many commercial and home-made games and toys and Apps on hand-held devices can help children acquire these prerequisite skills for mathematics learning and can better prepare them for all kinds of learning. To develop prerequisite skills successfully, games and toys should have certain characteristics:

1. Games should be based on strategies, not luck. In other words, to be proficient in a game should mean proficient in the game's strategies. This means that each encounter with the game or toy helps the child discover something more about the game, i.e., a strategy, a perspective, or a relationship between moves. Such games are interesting to novice and expert alike and help children improve their cognition, inquisitiveness, perseverance,

visualization, and executive functions.

2. In general, a game should last on average ten to fifteen minutes so that children can see the end of the game in a fairly short period of time. This helps them understand the relationship between a strategy and its impact on the game. This teaches children the foundation of deductive thinking or what can be understood as cause and effect. When a child has more interest and maturity and is able to handle delayed gratification, complex strategy games such as chess are meaningful.
3. Each game should develop at least one prerequisite mathematics skill. For example, the commercially available game Master Mind is an excellent means for developing pattern recognition and visual memory and for strengthening deductive thinking.

Every teacher and parent has a favorite list of games. Some games might have been prepared or collected for a specific purpose—reinforcing a skill, teaching a concept, strengthening a process, or just offering entertainment.

Following is a list of games and toys I have used extensively with children and adults to develop prerequisite skills and mathematics concepts and thinking. Most of these games and toys are commercial. They are highly motivational and can break formal instructional routines although they should not be used simply to occupy children's time. It is not exhaustive; it constantly changes. When I find a new game or a toy I examine it, use it with children. Identify the corresponding prerequisite skills. Sometimes I modify it and when it satisfies the following conditions I include it in this list.[2] The following games and toys satisfy the following conditions:

- have learning/educational value
- are fun and engaging
- are a natural activity in children's visual/perceptual development
- further cognitive, affective and psycho-motoric development
- are useful assessment tools

[2] I am always looking for new games and toys. If you come across a new game and want to discuss a game or a toy, please contact me at the Center (mahesh@mathematicsforall.org).

List of Games (with identified prerequisite skills)

- **Battleships (**spatial orientation, visualization, visual memory)
- **Black-Box** (logical deduction)
- **Blink** (pattern recognition, visual memory, classification, inductive reasoning)
- **British Squares** (spatial orientation, pattern recognition)
- **Card Games** (visual clustering, pattern recognition, number concept—visual clustering, decomposition/recomposition of number, number facts) (see Number War Games)
- **Checkers (**sequencing, patterns, spatial orientation/space organization)
- **Chinese Checkers** (patterns, spatial orientation/space organization)
- **Concentration** (visualization, pattern recognition, visual memory)
- **Cribbage (**number relationships, patterns, visual clusters)
- **Cross Number Puzzles** (number concepts, number facts)
- **Dominos (**visual clusters, pattern recognition, number concept and facts, decomposition/recomposition, number) (Number War Games)
- **Four Sight** (spatial orientation, pattern recognition, logical deduction)
- **Go Muko** (pattern recognition, spatial organization)
- **Go Make Ten** (**Go Fish Ten** or **Big Ten**) (number concept, decomposition/recomposition)
- **Hex** (pattern recognition)
- **In One Ear and Out the Other**[3] (number relationships, number facts, additive reasoning)
- **Kalah**, **Mankalah,** or **Chhonka** (sequencing, counting, estimation, visual clustering, deductive reasoning)
- **Krypto** (number sense, basic arithmetical facts)
- **Math Bingo Games** (number facts)
- **Master Mind** (sequencing, logical deduction, pattern recognition)
- **Number Master Mind** (number concept, place value, properties of numbers)

[3] Available from the Center.

- **Number Safari**[4] (number facts, additive and multiplicative reasoning, equations, a paper/pencil game)
- **Number War Games**[5] (visual clustering, arithmetic facts, mathematics concepts, deductive reasoning, fluency of facts)
- **Othello (**pattern recognition, spatial orientation, visual clustering, focus on more than one aspect, variable or concept at a time)
- **Parcheesi** (sequencing, patterns, number relationships)
- **Pinball Wizard**[6] (number facts, a paper/pencil game)
- **Pyraos** (spatial orientation/space organization)
- **Quarto** (spatial orientation/space organization, patterns, classification)
- **Qubic** (pattern recognition, spatial orientation, visualization, geometrical patterns)
- **Reckon** (number facts, estimation, basic operations)
- **Score Four** or **Connect Four** (pattern recognition, spatial orientation, visual clustering, geometrical patterns)
- **Shut the Box (sequencing, number concept, and number facts)**
- **Simon** or **Mini Wizard** (sequencing, following multi-step directions, visual and auditory memory)
- **Snakes and Ladders** (sequencing, following multi-step directions, visualization, number facts)
- **Stratego** (spatial orientation, logical deduction, graphing)

The key to the wise selection and use of games and toys is to have a range of games and toys available. Of course, it is useful to determine what prerequisite skills the child needs and then to select the appropriate games and toys. I have observed that once a child begins to get interested in games and toys, s/he is inclined to play with other games.

Categories of games that are universally enjoyed by children are card games related to number. The **Number War Games**[7] are based on the popular game ***Game of War*** using ordinary deck of cards to teach

[4] Available from the Center.
[5] Available from the Center.
[6] Available from the Center.
[7] The Descriptive Booklet (Games and Their Uses) available from the Center.

number and number relationships and to develop arithmetic skills. These games use ordinary decks of playing cards and dominos—a versatile set of tools for teaching mathematics from number conceptualization to introductory algebra. To help children develop number concepts, it is better to play these games with Visual Cluster Cards (cards without numbers on them, where the cluster on the card represents the number). Cards without numbers are available from the ***Center for Teaching/Learning of Mathematics.***

Number War Games are played essentially the same way as the Game of War and are easy to learn. To avoid the word war, you can call it by some other name such as: "beat it" or "top it." Children love to play these games. I have successfully used them for initial as well as remedial instruction, particularly for learning number, arithmetic facts, comparison of fractions, and operations on integers (treating club and spade cards as positive numbers and heart and diamond cards as negative numbers, for example, five of spades is $^{+}5$, and six of diamonds as $^{-}6$ and assign any value to face cards, e.g., Jack = 11, Queen = 12, King = 13, Joker = variable value, Ace = 1). Once they master arithmetic facts with these cards, one could extend the idea to algebra (e.g., In this game, one with bigger value for $P = 2x + 3y$, where x is the value of the red card and y is the value of the black card. The expression for P changes ($P = x^2 + y^2$, $P = 2x/3y$, $P = |x| - 3|y|$, *etc.)* with each game (See *Number War Games*[8] for detailed instructions).

In addition to developing prerequisite skills, games may be used in other ways:

- as a help in demonstrating the basic mathematical operations,
- to practice ease in computation,
- to improve processing speed,
- to gain speed and accuracy in recall.

Furthermore, games and play provide opportunities for discussions of strategies, outcomes, and feedback to improve strategies. Conversations invite children to communicate concepts while sharpening their thinking skills such as their ability to make inferences, to support their arguments with reasons, and to make analogies—skills essential to learning and applying mathematical skills.

In an environment where discussions are encouraged, children

[8] Available from *Center for Teaching/Learning of Mathematics*

begin to ask questions not only of their classmates and of siblings but also of parents. They learn to evaluate answers, draw conclusions, and follow up with more questions both of convergent (a question that calls for a yes, no or a short answer) and of divergent (a question that calls for an answer with explanation) types, which strengthen facility in reasoning. Use of reasoning is the core of mathematics learning.

Parents can do a lot in their children's education. Research shows that parental involvement—reading aloud, discussing the numbers children encounter in their environment, helping children to master arithmetic facts, checking homework, attending school meetings and events, setting expectations, relating current behavior with future accomplishments, and discussing school activities at home—has a more powerful influence on students' academic performance than anything about the school the students attend.

According to social science research, a major part of the academic advantage held by children from certain groups of families comes from well-organized and intentional cultivation of children and their interests. For this, parents do not need to buy expensive educational toys, take children to enrichment classes, or explore digital devices for their kids in order to give them an edge in academics. Rather, it is the discussion of ideas, exploration of interests, and support that develop children's interest and effort in learning.

The content and nature of these conversations and discussions also matters. Without discussions, children become procedurally oriented. But children who hear talk about quantity--counting and use of numbers at home start school with more extensive mathematical knowledge—more number words, comparative words, and sizes of numbers, relating numbers, and combining and breaking numbers apart—knowledge that predicts future achievement in mathematics. Similarly, discussions about the spatial aspects of their world have an impact on their understanding about the spatial properties of the physical world—how big or small or round, sharp objects, angles, or sides are. Both quantitative and spatial discussions give children's problem-solving abilities an advantage in future mathematics.

To the uninitiated, mathematical objects are abstract, unreal, but for those who enjoy mathematics they are real, almost concrete objects. Doing real mathematics is like playing a game; it is thinking about and acting upon mathematical objects and the relationships among them, using the same mental abilities that we use to think about physical space, other people, or games and toys. To engage children in mathematics and

excite them about mathematics learning, they need to see mathematics as a collection of interesting games and a means of communication. This communication is enhanced when there is an intentional effort to talk about mathematics to children.

Many of us feel comfortable talking about letters, words and sentences with our children—reading to them at night, talking about their games, toys, television programs, helping them decode their own books, noting messages on street signs and billboards. However, speaking to them about quantities (numbers, fractions, percents, decimals, greater, fewer, some, etc.), and shapes is not common.

Some parents engage their children in "math or number talk," but the amount and nature of number talk by parents with their children vary drastically. Studies show that early "number talk" at home is a key predictor of young children's achievement in mathematics once they get to school. The size of children's mathematics vocabulary, as in the case of native language, is dependent on exposure to the quantitative and spatial words. There are significant variations among families: Some children hear their parents speak only about two dozen number words a week, while others hear such words about 1,800 times weekly. There are gender differences in such talk as well. For example, many parents speak to their daughters about numbers far less than their sons, sometimes as little as half as much. Young children rapidly build their native language vocabularies and if encouraged, they can also build vocabularies for quantity and spatial concepts. Helping children become familiar with number and spatial concept words (to the left, above, below, next, farther, closer, bigger, higher, taller, longer, etc.) can promote their interest in mathematics as they enter school and in later grades.

The frequency of number talk in the children's homes has a big impact on how well the youngsters understand basic mathematical concepts such as the cardinal number principle, which holds that the last number reached when counting a set of objects determines the size of the set ("One, two, three—three apples in the bowl!") and then the idea that a number is the property of the collection and not just the outcome of the counting process. The kind of number talk that most strongly predicts later knowledge of numbers involves counting or labeling sets of objects that are right there in front of parent and child–especially large sets, containing between four and ten objects.

Though it may not come naturally at first, parents can develop the habit of talking about numbers as often as they talk about letters and words. Some ways to work numbers into the conversation are:

- Note numbers on signs when you're walking or driving with children, e.g., speed limits and exit numbers, building addresses, sale prices in store windows.
- Ask children to count how many toys they're playing with, how many books they've pulled out to read, or how many pieces of food are on their plate.
- Use numbers when you refer to time, dates, and temperatures: how many hours and minutes until bedtime, how many weeks and days until a holiday, the high and low the weatherman predicts for that day.
- With older children, math can become a part of talking about sports, science, history, video games, or whatever else they're interested in.

With practice, parents and children alike will find that math makes a very satisfying second language.

The Sequence of Strategies for Teaching Addition Facts[9]

Arithmetic facts are best developed and mastered, when the methods are based on decomposition/recomposition and using Visual Cluster Cards and Cuisenaire rods. Mastery means: (a) understanding/having a strategy, (b) fluency/automaticity, and (c) applicability. Addition facts (up to 20) should be mastered by the end of first grade, subtraction facts (up to 20) by the end of second grade, multiplication facts (10x10) by the end of third grade, and division facts (10x10) by the end of fourth grade.

0. Forty Five Sight facts using decomposition/recomposition
1 + 1; 1 +2, 2+1; 1+3, 2+2, 3+1; 1+4, 2+3, 3+2, 4+1; 1 +5, 2+4, 3+3, 4+2, 5+1; 1+6, 2+5, 3+4, 4+3, 5+2, 6+1; 1+7, 2+6, 3+5, 4+4, 5+3, 6+2, 7+1; 1+8, 2+7, 3+6, 4+5, 5+4, 6+3, 7+2, 8+1; 1+9, 2+8, 3+7, 4+6, 5+5, 6+4, 7+3, 8+2, 9+1. (Should be mastered at the end of Kindergarten.)

1. Commutative Property (The turn around facts) (M + N = N + M)
(*100* individual *facts are reduced to only 55 facts.)*
2. *N + 1, 1 + N (Adding one more to a number means getting the next number) [1+1; 1+2, 2+1; 3+1, 1+3; 1+4, 4+1; 5+1, 1+5; 1+6, 6+1; 1+7, 7+1; 8+1, 1+8; 9+1, 1+9; 10+1, 1+10]*
(19 facts on the Addition Grid: 10 on the first row and 9 on the first column in the grid.)
3. Making Ten (A pair of numbers that make 10?)
[2+8, 8+2; 3+7, 7+3; 4+6, 6+4; 5+5]
(7 new facts on the Addition Grid)
4. N + 10, 10 + N (The teens numbers)
[10+2, 2+10; 3+10, 10+3; 4+10, 10+4; 10+5, 5+10; 6+10, 10+6; 10+7, 7+10; 8+10, 10+8; 9+10, 10+9; 10+10]
(17 new facts on the Addition Grid)
5. N + 9, 9 + N (9 plus a number means adding 10 and minus 1)
[9+2, 2+9; 3+9, 9+3; 4+9, 9+4; 5+9, 9+5; 9+6, 6+9; 7+9, 9+7; 9+8, 8+9; 9+9] (15 new facts on the Addition Grid)
6. N + N (Double numbers)

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9 The most effective way of teaching Arithmetic facts is using Cuisenaire rods (see Cuisenaire rods and Mathematics Learning by Mahesh Sharma, 1988)

[2+2, 3+3, 4+4, 6+6, 7+7, 8+8]
(6 new facts on the Addition Grid)
7. N + (N+1) and (N + 1) + N (Near Doubles; If I know the double, I know one more than the double or one less than the double)
[2+3, 3+2; 3+4, 4+3; 4+5, 5+4; 5+6, 6+5; 6+7, 7+6; 7+8, 8+7]
(12 new facts on the Addition Grid)
8. N + (N − 2) (Adding Numbers that are two apart, then the their sum = double of the middle) [2+4, 4+2; 3+5, 5+3; 5+7, 7+5; 6+8, 8+6]
(8 new facts on the Addition Grid)
9. N + 2, 2 + N (Adding 2 to a number means skipping a number)
[5+2, 2+5; 6+2, 2+6; 7+2, 2+7]
(6 new facts on the Addition Grid)
10. Near Tens (The sum of two numbers, that is 1 more or 1 less than 10) [3+6, 6+3; 4+7, 7+4; 3+8, 8+3]
(6 new facts on the Addition Grid)
11. Remaining four facts
[8+4, 4+8; 8+5, 5+8]

The Sequence Strategies for Teaching Multiplication Facts

1. Commutative Property (Turn around facts: M · N = N · M)
(The 100 individual facts of multiplication are reduced to only 55 facts.)
2. N · 1 = 1 · N (Table of 1: Multiplying a number by one results in the same number)
[1×1; 1×2, 2×1; 3×1, 1×3; 1×4, 4×1; 5×1, 1×5; 1×6, 6×1; 1×7, 7×1; 8×1, 1×8; 9×1, 1×9; 10×1, 1×10]
(19 individual facts on the Multiplication Grid)
3. N × 10, 10 × N (Table of 10 means counting by 10)
[10×2, 2×10; 3×10, 10×3; 4×10, 10×4; 10×5, 5×10; 6×10, 10×6; 10×7, 7×10; 8×10, 10×8; 9×10, 10×9; 10×10]
(17 new individual facts on the Multiplication Grid)
4. N × 5, 5 × N (Table of 5 means counting by 5)
[5×2, 2×5; 5×3, 3×5; 5×4, 4×5; 5×5; 5×6, 6×5; 5×7, 7×5; 5×8, 8×5; 9×5, 5×9]
(15 new individual facts on the Multiplication Grid]
5. **N × 2, 2 × N (Table of 2 means doubles or counting by 2**)
[2×2; 2×3, 3×2; 2×4, 4×2; 2×6, 6×2; 2×7, 7×2; 2×8, 8×2; 9×2, 2×9]
(13 new individual facts on the Multiplication Grid)
6. N × 9, 9 × N
[3×9, 9×3; 4×9, 9×4; 9×6, 6×9; 7×9, 9×7; 9×8, 8×9; 9×9]
(11 new individual facts on the Multiplication Grid)
7. 4 × N, N × 4
[4×3, 3×4; 4×4; 4×6, 6×4; 4×7, 7×4; 8×4, 4×8]
(9 new individual facts on the Multiplication Grid)
8. N × N (Square numbers)
[3 × 3, 6 × 6, 7 × 7, 8 × 8]
(4 new facts on the Multiplication Grid)
9. N × 3, 3 × N (Table of 3)
[3 × 6, 6 × 3; 3 × 7, 7 × 3; 3 × 8, 8 × 3]
(6 new facts on the Multiplication Grid]
10. The remaining six facts [8 × 6, 6 × 8; 8 × 7, 7 × 8; 7 × 6, 6 × 7]
11. Distributive property: 12 × 8 = (10 + 2) × 8 = 10 × 8 + 2 × 8.

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	arithmetic facts	basic operations	classification.	concept of Time	counting	estimation	focus on many concepts	following multistep directions	geometrical patterns	graphing	logical deduction	mathematics concepts	number facts	number relationships	number sense	paper/pencil	pattern recognition	patterns	sequencing	space organization	spatial orientation	visual and auditory memory	**visual clusters**	**visual memory**	**visualization**
Simon or Mini Wizard								◙											◙			◙		◙	
Battleships																			◙					◙	◙
Cribbage			◙															◙		◙	◙				
Quarto																	◙								◙
Concentration																		◙						◙	◙
Chinese Checkers																		◙		◙	◙				
Pachisi														◙				◙	◙						
Checkers																		◙	◙	◙	◙				
Othello				◙			◙										◙			◙			◙		

Score Four/Connect Four									◙								◙				◙		◙		
Qubic									◙									◙			◙				◙
Pyraos																				◙	◙				
Krypto	◙														◙										
Go Muko																	◙			◙					
Kalah or Mankalah					◙	◙													◙				◙		
Master Mind											◙						◙		◙						
Four Sight											◙						◙			◙					
Black- Box											◙														
Reckon		◙					◙																		
Card Games													◙					◙					◙		
Dominos													◙					◙					◙		
Hex																					◙				
British Squares																					◙				
Stratego										◙	◙										◙				
Number Safari													◙			◙									
Pinball Wizard													◙			◙									
In One Ear & Out the Other													◙			◙									
Number War Games	◙											◙												◙	

Center for Teaching/Learning of Mathematics

CT/LM has programs and materials to assist teachers, parents, tutors, and diagnosticians to help children/adults with their learning difficulties in mathematics. We conduct regular **workshops, seminars,** and **lectures** on topics such as:

1. How does one learn mathematics? This workshop focuses on psychology and processes of learning mathematics—concepts, skills, and procedures. The role of factors such as: Cognitive development, language, mathematics learning personality, pre-requisite skills, conceptual models, and key developmental milestones (number conceptualization, place value, fractions, integers, algebraic thinking, and spatial sense) in mathematics learning. Participants learn strategies to teach their students more effectively.

2. What are the nature and causes of learning problems in mathematics? This workshop focuses on understanding the nature and causes of learning problems in mathematics. We examine existing research on diagnosis, remedial and instructional techniques in dealing with these problems. Participants become familiar with diagnostic and assessment instruments for learning problems in mathematics. They learn strategies for working more effectively with children and adults with learning problems in mathematics such as: dyscalculia and math anxiety.

3. Content workshops. These workshops are focused on teaching key mathematics milestone concepts and procedures. For example, **How to teach arithmetic facts easily and effectively. How to teach fractions more effectively. How to develop the concepts of algebra easily. Mathematics As a Second Language.** In these workshops, we use a new approach called **Vertical Acceleration.** In this approach, we begin with a simple concept from arithmetic and take it to the algebraic level.

4. What to look for in a results-oriented mathematics classroom: This is a workshop for administrators and teachers to understand the key elements necessary for an effective mathematics classroom.

We offer **individual diagnosis** and **tutoring services** for children and adults to help them with their mathematics learning difficulties, general learning problems, and dyscalculia. We provide:

1. Consultation with and training for parents and teachers to help their children cope with and overcome their anxieties and difficulties in learning mathematics, including dyscalculia.
2. Consultation services to schools and individual classroom teachers to help them evaluate their mathematics programs and teaching and help design new programs or supplement existing ones in order to minimize the incidence of learning problems in mathematics.
3. Assistance for the **adult student** who is returning to college and has anxiety about his/her mathematics.
4. Assistance in test preparation **(SSAT, SAT, GRE, GMAT, MCAS**, etc.)
5. Extensive array of mathematics publications to help teachers and parents to understand how children learn mathematics, why learning problems occur and how to help them learn mathematics better.

Publications

Dyslexia and Mathematics Language Difficulties	**$15.00**
How to Master Arithmetic Facts Easily and Effectively	**$15.00**
Guide for an Effective Mathematics Lesson	**$15.00**
How to Teach Fractions Effectively	**$15.00**
Math Education at Its Best: Potsdam Model	**$15.00**
How to Teach Subtraction	**$12.00**
Literacy & Numeracy: Comprehension and Understanding	**$12.00**
How to Teach Number to Young Children	**$15.00**
Dyscalculia	**$15.00**
The Games and Their Uses in Mathematics Learning	**$12.00**
The Questioning Process: The Basis of an Effective Lesson	**$12.00**
Playing cards without numbers	**$12.00**

DVDs

How Children Learn: Numeracy **$30.00**

(One-hour long video interviewing Professor Mahesh Sharma on his ideas about how children learn mathematics)

How To Teach Place Value **$30.00**

(Strategies to teach place value effectively)

How Children Learn Numeracy

(Complete set of six DVDs for **$150.00** and individual for **$30.00**)

1. **Teaching arithmetic facts,**
2. **Teaching place value,**
3. **Teaching multiplication,**
4. **Teaching fractions,**
5. **Teaching decimals and percents, and**
6. **Professional development: Teachers' questions**

Most children have difficulty in mathematics when they have not mastered the key mathematics milestones in mathematics. The key milestones for elementary grades are: Number conceptualization and arithmetic facts (addition and multiplication), place value, fractions and its correlates—decimal, percent, ratio and proportion. These videos and DVDs present strategies for teaching these key mathematics milestone concepts. They apply Prof. Sharma's approach to teaching numeracy. These were videotaped in actual classrooms in the UK**.**

Please mail or fax order (add 20% extra for postage & handling) to:

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Mahesh Sharma
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Professor Mahesh Sharma is the founder and President of the Center for Teaching/Learning of Mathematics, Inc., Framingham, Massachusetts, and Berkshire Mathematics in England. Berkshire Mathematics facilitates his work in the UK and Europe.

He is the former President and Professor of Mathematics Education at Cambridge College, where for more than thirty-five years, he taught mathematics and mathematics education to undergraduate and graduate students.

He is internationally known for his groundbreaking work in mathematics learning problems and mathematics education, particularly dyscalculia and other specific learning disabilities in mathematics. He is an author, teacher and teacher-trainer, researcher, consultant to public and private schools, as well as a public lecturer.

Professor Sharma was the Chief Editor and Publisher of *Focus on Learning Problems in Mathematic*s, an international, interdisciplinary research journal with readership in more than 90 countries, and the Editor of *The Math Notebook*, a practical source of information for parents and teachers devoted to improving teaching and learning for all children.

Professor Sharma provides direct services of evaluation and tutoring for students (children as well as adults) who have learning disabilities such as dyscalculia or face difficulties in learning mathematics and gifted/talented children to help them reach their potential. He works with teachers and school administrators to design strategies to improve mathematics curriculum and instruction for all. He has been a consultant to many educational organizations, school systems, states and provinces in North America, and countries in Asia and Europe. He has at several universities in the USA, Canada, UK, and Asia.

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